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SciVerse ScienceDirect

00 (2015) 000–000

Procedia Engineering

www.elsevier.com/locate/procedia

24th International Meshing Roundtable (IMR24)

High-order unstructured curved mesh generation using the Winslow equations

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Abstract

We propose a method to generate high-order unstructured curved meshes using the classical Winslow equations. We start with an initial straight-sided mesh in a reference domain, and fix the position of the nodes on the boundary on the true curved geometry. In the interior of the domain, we solve the Winslow equations using a new continuous Galerkin finite element discretization. This formulation appears to produce high quality curved elements, which are highly resistant to inversion. In addition, the corresponding nonlinear equations can be solved efficiently using Picard iterations, even for highly stretched boundary layer meshes. Compared to several previously proposed techniques, such as optimization and approaches based on elasticity analogies, this can significantly reduce the computational cost while producing curved elements of similar quality.

Peer-review under responsibility of organizing committee of the 24th International Meshing Roundtable (IMR24).

Keywords: Unstructured high-order meshes, curved mesh generation, Winslow equations

1. Problem formulation and discretization

Let $D \subset \mathbb{R}^n$ and $C \subset \mathbb{R}^n$ denote the physical and the computational domain, respectively. Define the mapping $\boldsymbol{x} : C \to D$, where $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{\xi}) = (x_1(\boldsymbol{\xi}), \dots, x_n(\boldsymbol{\xi}))$. The Winslow equations in physical coordinates are given by:

$$g^{ij}\partial_i\partial_j x_k = 0 \quad \text{for } k = 1, \dots, n, \tag{1}$$

where, g^{ij} are defined through the relation $g^{ij}g_{jk} = \delta_{ik}$ and $g_{ij} = \partial_i x_k \partial_j x_k$.

Assuming sufficient smoothness of the solution fields, we can then rewrite Eqs. (**??**) as a conservative second-order term plus a first order term involving $\boldsymbol{\alpha}$, to obtain the final form of our governing equations:

$$\begin{aligned} \partial_i(g^{ij}) + \alpha_j &= 0, && \text{for } j = 1, \dots, n, \\ \partial_i(g^{ij}\partial_j x_k) + \alpha_j \partial_j x_k &= 0, && \text{for } k = 1, \dots, n. \end{aligned}$$

Our discretization using standard continuous Galerkin method lead to the final system of equations with nonlinear dependencies:

$$M\alpha_j^h = b_j(\boldsymbol{x}^h), \qquad j = 1, \dots, n, \tag{2}$$

$$K(\boldsymbol{\alpha}^h, \boldsymbol{x}^h)x_j^h = c_j(\boldsymbol{\alpha}^h, \boldsymbol{x}^h), \qquad j = 1, \dots, n. \tag{3}$$

We solve these nonlinear equations using Picard iterations. Namely, for a given solution iterate $\boldsymbol{x}^{(\ell)}$, we compute an improved iterate by the following steps

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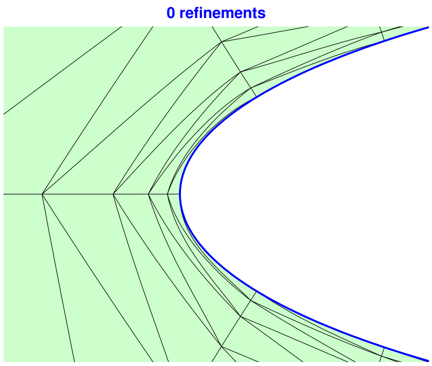
1. Assemble (**??**) using $\boldsymbol{x}^h = \boldsymbol{x}^{(\ell)}$ and solve for $\boldsymbol{\alpha}^h = \boldsymbol{\alpha}^{(\ell)}$.
2. Assemble (**??**) using $\boldsymbol{x}^h = \boldsymbol{x}^{(\ell)}$ and $\boldsymbol{\alpha}^h = \boldsymbol{\alpha}^{(\ell)}$, and solve for $\boldsymbol{x}^{(\ell+1)}$.

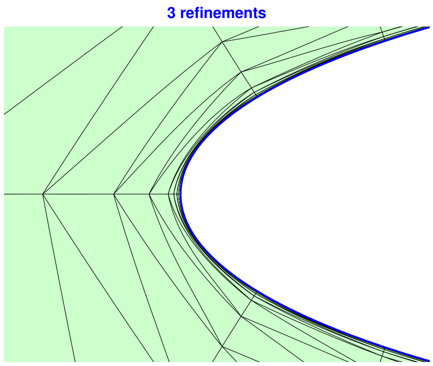
2. Results

Here we present the results our method for two examples: an anisotropic mesh with boundary layers and a Falcon aircraft configuration (Fig. **??**).

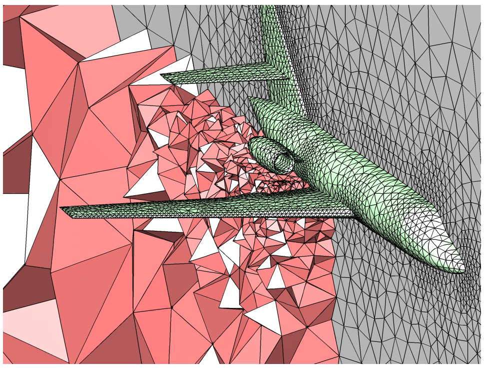
We also study the behavior of our solver as we locally refine the mesh close to the boundary layer with growth factor equal to 2. We count the number of iterations the method takes to converge, and observe that it remains mainly constant as we refine the mesh – in contrast with non-linear elasticity approach [**?**] where it scales by the inverse of the thickness of the boundary layer.



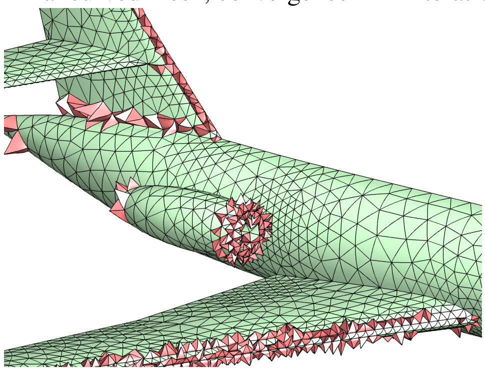
(a) Final curved mesh, convergence in 10 iterations



(b) Final curved mesh, convergence in 11 iterations



(c) Final curved mesh, Falcon aircraft



(d) Final curved mesh, Falcon aircraft, elements with scaled Jacobian smaller than 0.5

Fig. 1. Refinement study: In (a) and (b), show local refinement pattern, growth factor is 2. In (c) and (d), we see the Falcon aircraft configuration: the minimum scaled Jacobian for this example was 0.20.

References

[1] Per-Olof Persson and Jaime Peraire, Curved mesh generation and mesh refinement using lagrangian solid mechanics, Proceedings of the 47th AIAA Aerospace Sciences Meeting and Exhibit, 2009.